Rivulet Structures in Falling Liquid Films

B. Scheid

1 First Visit at the Instituto Pluridisciplinar

April 1st 1998 was the day I started my Ph.D. thesis (not a joke!) supervised by Jean Claude Legros and Pierre Colinet. I do not remember when exactly I started to hear about Manuel G. Velarde (the G. is for García that should never be omitted!), but I bet it was very soon after the beginning. Pierre was finishing his postdoc with Manuel and he always looked very happy (and somehow tired) each time he came back from Madrid. So one day, I had the opportunity to visit a friend who lived there and I decided to stop by the Instituto Pluridisciplinar to check by myself what was so unique with this place. Of course my visit was coordinated by Pierre and Manuel. The goal was to meet Jan Skotheim and Uwe Thiele and discuss about the experiments I was doing at that time in Brussels with Oleg A. Kabov. Jan was a student coming from MIT and Uwe was a postdoc. We talked about rivulet structures arising in localized heated falling liquid films, as shown in Fig. 1. This collaboration lead us to a nice JFM paper on the stability analysis of a bump arising at the upper edge of the heater prior to the rivulet instability [1].

I always remember the analogy Manuel did between rivulet formation and crowd dynamics: if you have a group of people running in one direction (the hydrodynamic flow) and another group of people running in the perpendicular direction (the thermocapillary flow), we will soon see the emergence of auto-organized structure (rivulets) in which people will preferentially run to minimize their loss of energy…

That was my first experience at the Instituto Pluridisciplinar.

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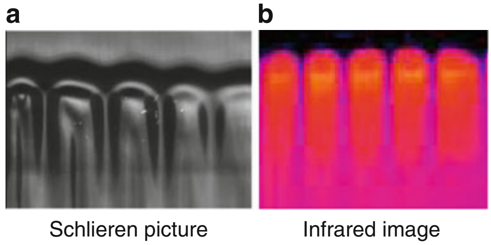

Fig. 1 Experimental images showing rivulet structures arising in a locally heated falling liquid film: the liquid arrives on a heated zone such as its surface tension decreases. A thermocapillary stress drives liquid along the interface against gravity, what produces a bump. Above criticality, this bump becomes unstable and breaks into rivulets as for the tears of wine

2 A Great Experience in Madrid

January 2002, I arrived in Madrid to work on my Ph.D. thesis with Manuel in the frame of the Marie-Curie European Network ICOPAC. I was told that Manuel usually prefers hiring postdocs than Ph.Ds. so I had to prove him he made the right choice. I did not know what to expect exactly but I felt like it was a great opportunity for me to work there. I took the office in which Jan and Uwe were sited 2 years earlier. It was facing the mountains around Madrid, very inspiring! On my desk was the thesis of Christian Ruyer-Quil. I didn't know about his work before but soon after I opened it, I understood that it was very important for the course of my own Ph.D. The first discussion I had with Manuel on the blackboard lasted only 5 min. He asked me to reschedule a meeting after changing the way I was presenting my problem. I had to use arguments! All the other meetings lasted much longer…

We were modeling wave dynamics in heated falling liquid films, with the aim to understand the occurrence of rivulet structure, not only with localized heating but also in the more general case of uniform heating. Since Manuel and Christian had already started discussing that problem, I naturally went to the FAST laboratory for 6 months with another Marie-Curie fellowship, before I came back to Madrid for 6 more months in 2003. That was probably one of the most stimulating years of my scientific career; traveling between Paris, Madrid and Brussels.

In August 2003, Manuel organized a crucial meeting with Christian, Serafim Kalliadasis and Philip Trevelyan. We spent a week during which I presented all the results of my Ph.D. that I was about writing. By the end of the week, we decided to prepare a book all together. That was the beginning of another great adventure.

3 The Book

Initially, we thought that the book could be finalized in about 2 years. It was published in 2012 [2]! So you never know, but what is sure is that Manuel was the driving force, putting us together several times in Madrid for 1 or 2 weeks between 2003 and 2006. I keep an incredible souvenir of the productive time spent at the Instituto with Manuel, Christian and Serafim. The experience of working simultaneously on a joint project is unique and the quiet atmosphere at the Instituto plus the Madrileñan climate surely helped significantly.

The last chapter of the book is precisely about the modeling of uniformly heated falling liquid film in 2D and in 3D. Let us now recall the main step of this modeling, since it is representative of the joined effort we have all put together during this fruitful Madrileñan collaboration.

We proposed a model of four evolution equations for the film thickness h, the streamwise and transverse flow rates averaged across the film, q and p respectively, and the interfacial temperature θ, all dependent only on time t and in-plane coordinates (x,z), with x along the main flow and z in the spanwise direction. Slow time and space modulations of the basic flat-film state have been assumed, namely $\partial_t, \partial_x, \partial_z \sim \epsilon \ll 1$ where ϵ is an ordering parameter. Having posed self-similar profiles in terms of the natural similarity variable $\overline{y} = y/h$ (y being the cross-stream coordinate), namely a parabolic profile for the velocity $f_0(\overline{y}) = \overline{y} + (1/2)\overline{y}^2$ and a linear profile for the temperature $g_0(\overline{y}) = \overline{y}$, the model has been obtained by averaging along the thickness the momentum and energy equations with weights taken equal to f_0 and g_0, respectively, like in the Galerkin method (details of the procedure and results are given in [3, 4] for the 2D case). Provided the no-slip condition applies on the wall and the viscous stresses balancing the (thermo)capillary effects at the free surface, the average momentum equation at order ϵ, for the 3D case, has the form

$$\partial_t \mathbf{q} = \frac{5}{6} h\,\mathbf{i} - \frac{5}{2}\frac{\mathbf{q}}{h^2} - \frac{5}{4}\mathrm{Ma}\boldsymbol{\nabla}\theta + \frac{5}{6}\Gamma h \boldsymbol{\nabla}(\boldsymbol{\nabla}^2 h) + \frac{5}{6}\mathrm{Ct}\, h \boldsymbol{\nabla} h + \frac{9}{7}\left(\frac{\mathbf{q}\cdot\boldsymbol{\nabla}h}{h^2} - \frac{\mathbf{q}}{h}\cdot\boldsymbol{\nabla}\right)\mathbf{q} - \frac{8}{7}\frac{\boldsymbol{\nabla}\cdot\mathbf{q}}{h}\,\mathbf{q}\,, \tag{1}$$

where $\boldsymbol{\nabla} = (\partial_x, \partial_z)$, $\mathbf{q} = (q, p)$ and $\mathbf{i}$ is the streamwise unit vector. The first five terms of the r.h.s. account for the gravity acceleration, viscous shear stress, Marangoni effect, surface tension and hydrostatic pressure, respectively, and the remaining terms are due to inertia. Then, provided the temperature field is uniform at the wall and satisfies the Newton's law of cooling at the free surface, the average energy equation at order ϵ has the form

$$\partial_t \theta = 3\frac{(1-\theta-\mathrm{Bi}h\theta)}{\mathrm{Pr}\,h^2} + \frac{7}{40}(1-\theta)\frac{\boldsymbol{\nabla}\cdot\mathbf{q}}{h} - \frac{27}{20}\frac{\mathbf{q}\cdot\boldsymbol{\nabla}\theta}{h}\,, \tag{2}$$

where the first term of the r.h.s accounts for the in-depth heat transfer and the remaining terms are due to heat convection. The system is closed by mass conservation

$$\partial_t h + \boldsymbol{\nabla}\cdot\mathbf{q} = 0\,. \tag{3}$$

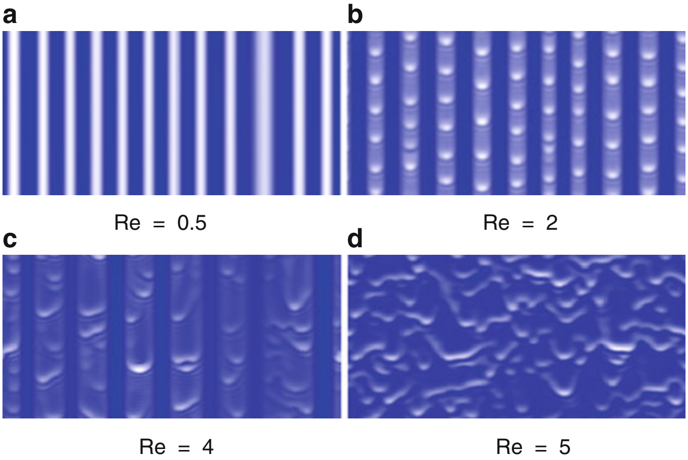

Fig. 2 Simulated wave patterns arising in uniformly heated falling liquid films for various Reynolds numbers. Water properties have been used, with $\Gamma = 3,375$, $\mathrm{Ct} = 0$, $\mathrm{Ma} = 25$ and $\mathrm{Bi} = 0.1$. Snapshots are right before rupture, except for (**d**)

The length scale and time scale are $\ell_\nu = (\nu^2/g \sin\beta)^{1/3}$ and $t_\nu = (\nu/g \sin\beta^2)^{1/3}$, with ν the kinematic viscosity and g the gravitational acceleration. The governing dimensionless groups are the Kapitza number $\Gamma = \sigma\ell_\nu/\rho\nu^2$, the inclination number $\mathrm{Ct} = \cot\beta$, the Marangoni number $\mathrm{Ma} = \gamma\Delta T\ell_\nu/\rho\nu^2$, the Biot number $\mathrm{Bi} = \alpha\ell_\nu/k$ and the Prandtl number $\mathrm{Pr} = \nu/\chi$, with ρ the density, σ the surface tension and $\gamma = -\mathrm{d}\sigma/\mathrm{d}T$ its variation with temperature, β the inclination angle of the wall from the horizontal, ΔT the temperature difference between the wall and the ambient gas, α the heat transfer coefficient at the liquid–gas interface, k the thermal conductivity and χ the thermal diffusivity. The Reynolds and Weber numbers appear implicitly through the thickness of the uniform flat film h_N as $\mathrm{Re} = g\sin\beta h_N^3/3\nu^2$ and $\mathrm{We} = \sigma/\rho g \sin\beta h_N^2$. All numbers are taken of $O(1)$, except $\Gamma \sim \mathrm{We} = O(\epsilon^{-2})$. For isothermal conditions ($\mathrm{Ma} = 0$), the present model reduces to the $O(\epsilon)$-version of the 3D model given in [5], validated against both DNS and experiments. Notice that the coefficients in the momentum and energy equations differ from unity because of the non-uniformity of the base state profiles, precisely represented by f_0 and g_0. These coefficients are necessary to recover the Benney-like equation (see e.g. [6]) through a gradient expansion of the present model in the limit $\mathrm{Re} \ll 1$.

In the case of vertical wall, we have performed simulations whose results are shown in Fig. 2 (see [7, 8] for details). For small Reynolds number ($\mathrm{Re} = 0.5$), the hydrodynamic wave amplitude is negligible as compared to the amplitude of rivulets induced by thermocapillary flow, while for large Reynolds number ($\mathrm{Re} = 5$) the hydrodynamic waves dominate the system and prevent the formation of rivulets. For intermediate Reynolds numbers, there is a region where inertia (hydrodynamic) and thermocapillary instabilities are equally important. In this region, we found the appearance of large solitary waves channeled by rivulets aligned with the flow.

We were very excited by these results, especially as fluid flow settings where 2D pulses can be stabilized are quite rare. And everyone knows about the interest of Manuel for solitary pulses…

4 Rivulets with Negative Gravity

We also believed that the novelty of the resulting pattern showed in Fig. 2 might very well be generic for systems exhibiting a competition between monotonic and oscillatory (or wave) instabilities with anisotropy (here, due to the direction of the basic flow). In fact another competition of instabilities, embedded in (1), is also possible in case of negative gravity. This is considering an isothermal liquid film flowing underneath an inclined wall. In such a case, the dimensionless number Ct becomes negative, meaning that the hydrostatic pressure plays a destabilizing role, exactly like in the Rayleigh–Taylor instability that breaks a condensed film on a ceiling into an array of droplets. The difference with the pure Rayleigh–Taylor instability is that the flow breaks the symmetry and thus promotes the formation of rivulets, as shown in Fig. 3.

The rivulet structures shown in Fig. 3 can only be obtained for a given range of inclination numbers. This range can be identified by tuning the inclination number such that the contribution of the Rayleigh–Taylor instability to the growth rate of small perturbations is of the same order of magnitude than the contribution of the thermocapillary instability in the case of heated falling film shown in Fig. 2. This argument is inferred from the linear stability analysis of the uniform-film base-state of the system of equations (1)–(3). It consists in imposing a small harmonic disturbance writing

$$h = \overline{h}_N + \eta \exp\{\mathrm{i}\,(k\,x - c\,t) + s\,t)\}\,, \tag{4}$$

where η, k, c and s are real numbers and represent, respectively, the amplitude, the wavenumber, the phase speed and the growth rate of the disturbance; $\overline{h}_N = h_N/\ell_\nu$ is the dimensionless flat film thickness. Inserting this "normal mode representation" (4) into the two-dimensional form of (1)–(3) and linearizing in η yield the linear phase speed and growth rate:

$$c_\mathrm{L} = \overline{h}_N^2 \quad \text{and} \quad s = k^2 \overline{h}_N^3 \left(\frac{2}{15}\overline{h}_N^3 - \frac{1}{3}Ct + \frac{1}{2}\frac{\mathrm{Bi\,Ma}}{\overline{h}_N(1+\mathrm{Bi}\overline{h}_N)^2} - \frac{1}{3}\Gamma, k^2 \right). \tag{5}$$

The surface waves will grow for $s > 0$, i.e. for disturbance wavenumbers smaller than the critical (cut-off) wavenumber k_c obtained for $s = 0$. In (5), one can see that a negative Ct will be destabilizing with the same contribution to the growth rate than in the case of a vertical heated wall if $Ct \sim$ BiMa, provided $\overline{h}_N \sim 1$. The angle we finally found this way to obtain rivulet structures is $\beta = 160°$, or Ct $= -2.75$.

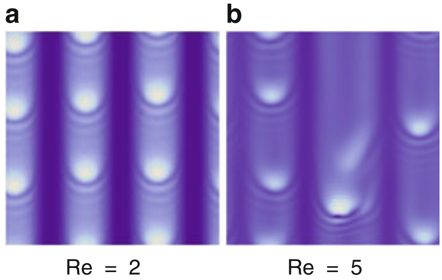

Fig. 3 Simulated wave patterns arising in falling liquid films flowing underneath an inclined wall. Water properties have been used, with $\Gamma = 3,375$, $\mathrm{Ct} = -2.75$ and $\mathrm{Ma} = 0$

Concerning the dynamics of the wave pattern consisting of solitary pulses riding rivulets, we found a fundamental difference between the rivulets formed by thermocapillary effects and those formed by negative gravity. In the former case, the rivulets grow until the thin film in between ruptures, while in the latter case, rivulet growth seems to saturate and yields stabilized pattern. This is at least what we observe for Re $=$ 2, in Fig. 3a, namely that the wave pattern did not change during a relatively long time. However, for Re $=$ 5, in Fig. 3b, the largest wave hump becomes too heavy to saturate and instead detaches after some time. No stable structure were thus observed in this case. These behaviors and the mechanism of stabilization are subject to further investigations but the aim was too show here that the field of 3D wave pattern in falling films under several configurations is still an active field of research, as illustrated for instance by a recent paper, precisely on falling films flowing down inverted substrates [9].

5 Marvelous Friendships

The aim of the present article was to show how influential has been the role of Manuel in my research activity for more than 10 years. It was not only through direct collaborations and discussions with him but also through the numerous collaborations that he has encouraged, driven by his enthusiasm to put people together in order to solve new problems. The result can not only be measured by the number of joint papers but mostly by the marvelous friendships that come out of all these collaborations. I'm thus infinitely grateful to Manuel for his indefectible support he always reserve to his friends in general and to me in particular.

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