STATIONARY THERMOCAPILLARY INSTABILIITY IN A THIN FERROFLUID LAYER SUBMITTED TO A WEAK MAGNETIC FIELD

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ABSTRACT We study the thermocapillary instability in a thin horizontal ferrofluid layer with a deformable free surface. The magnetic liquid is submitted to a weak magnetic field and a thermal gradient, both normal to the layer boundaries. Two different types of instability may occur in the liquid. In the isothermal case, static Rosensweig instability may exit due to the magnetic force acting on the free surface. When the layer is heated, variations in the surface tension along the liquid-gas surface, due to temperature gradients, induce Marangoni convection in the bulk. In the present paper, we study the necessary conditions for the appearance of both instabilities for the non-inductive case. The linear stability analysis is applied in the case of stationary perturbations and the compatibility condition is obtained in two equivalent analytical forms representing either the Marangoni number or the magnetic Bond number as a function of the wavenumber. Previous results from other authors for thermocapillary instability in a ferrofluid layer are revised and corrected.

INTRODUCTION

As was shown by Cowley & Rosensweig [1] (see also Rosensweig's book [2]), when a magnetic field is applied perpendicularly to a free surface of a quiescent liquid, the surface becomes unstable beyond a critical value of the magnetic field and many peaks appear on it, giving rise to a new static shape. This specific instability is due to the balance between the magnetic force, the surface tension and the gravity force. For an inviscid isothermal ferrofluid filling a semi-infinite space, the critical values of the magnetization M and the wavenumber k are given by [1, 2]

$$M_{crit}^{2} = \frac{2}{\mu_0}\left(1+\frac{\mu_0}{\mu}\right)\sqrt{\rho\sigma g} \qquad \text{and} \qquad k_{crit} = \sqrt{\frac{\rho g}{\sigma}}, \tag{1}$$

where $\mu(\mu_0)$ is the magnetic permittivity of the ferrofluid (respectively, of vacuum), ρ the liquid density, σ the surface tension and g the acceleration of gravity. Stability of the free surface of inviscid isothermal ferrofluids and surface waves were also studied in [3]. Later, finite width layers of viscous magnetic liquids with one [4] or two [5] free boundaries were considered (see details in [6] and [7]). The influence of the layer depth on the critical magnetic field was investigated in [8].

In a thin layer of ferrofluid, submitted to a vertical temperature gradient, the Benard-Marangoni instability can occur when the magnitude of the gradient becomes sufficiently large, like in any usual Newtonian fluid. Under the combined action of both magnetic and temperature fields a coupling between the Rosensweig and the Marangoni instability is expected. The interfacial instability of a layer of thermally conducting ferrofluid was first considered by Bashtovoi and Krakov [9]. Neglecting all bulk forces and the convective heat transfer, they obtained the compatibility condition representing the Marangoni number as a function of the wavenumber and containing a (magnetization) parameter which stands for the surface magnetic instability. This equation was simplified for small and large wavenumbers. The stationary thermocapillary instability was also studied for layers on thermally conducting [10] and insulated [11] solid walls, taking into account the convective heat transfer. In the last three papers heating from below leads always to an unstable situation while heating from above would generate instability if the temperature gradient exceeds certain threshold value. The experiments of Schwab et al. [12, 13] however showed that an unstable pattern would emerge whatever the direction of the heat flux.

Unaware of the earlier works [9, 10, 11], Weilepp and Brand [14] studied the coupling between the Rayleigh- Bénard-Marangoni and the Rosensweig instability for a finite depth layer, neglecting the Kelvin force in the bulk momentum balance. Thus, the magnetic field influence reduces to the magnetic traction acting on the free surface. These authors considered only heating from *below* for both oscillatory and non-oscillatory disturbances.

Here we re-formulate the thermocapillary instability problem considering non-oscillatory disturbances only. We restrict ourselves to the non-inductive case. The linear stability analysis leads to the compatibility condition written in two different, but equivalent forms, one of which corresponding to that of Pavlinov [10] and the other to that of Weilepp and Brand [14]. An asymptotic expression of the compatibility condition is derived for very small wavenumbers and compared with that used in the literature [6, 10].

FORMULATION OF THE PROBLEM

Consider a horizontal ferrofluid layer of width d and of infinite lateral extent, bounded by a non-magnetic solid wall from the one side and open to a gaseous, magnetically inert phase from the other side. The magnetic liquid is submitted to a weak magnetic field H^{ext} and to a temperature gradient β, both normal to the layer boundaries. Cartesian coordinates (x, y, z) are introduced with axes x and y

along the wall plane and axis z directed from the wall towards the liquid-gas surface, that last being flat in the reference unperturbed state. Relatively to the gravity field the layer can rest above the wall plane or it is hanging from the ceiling. Considering only thin layers with a deformable free surface, we disregard the buoyancy effect, but take into account the weight of the liquid above or below the unperturbed flat position of the surface.

The ferrofluid is considered as a heat conducting Newtonian liquid with constant physical properties (density, viscosity and thermal conductivity), except the surface tension assumed to be a linear function of the temperature T :

$$\sigma = \sigma_0 \left[1 - \gamma\left(T - T_{ref}\right)\right], \qquad \gamma = -\frac{1}{\sigma_0}\frac{d\sigma}{dT} > 0, \tag{2}$$

where σ_0 is a value at the reference liquid-gas surface temperature T_{ref} and γ is the surface tension coefficient.

Maxwell equations

The ferrofluid is described as a super paramagnetic gas [2, 6]. It is assumed that in the layer the magnetization $\mathbf{M}$ is collinear to the magnetic field

$$\mathbf{M} = M\,\mathbf{l}_H, \qquad \mathbf{M} = \frac{M}{H}\mathbf{H} = \chi\,\mathbf{H}, \qquad \mathbf{l}_H = \frac{\mathbf{H}}{\sqrt{\mathbf{H}\cdot\mathbf{H}}}. \tag{3}$$

The susceptibility χ is constant for weak magnetic fields. The Maxwell equations for the magnetic field intensity and the magnetic conductivity $\mathbf{B} = \mu_0\left(\mathbf{H} + \mathbf{M}\right) = \mu_0\left(1+\chi\right)\mathbf{H}$ of ferrofluids are written as

$$\nabla \times \mathbf{H} = 0, \qquad \nabla\cdot\mathbf{B} = 0. \tag{4}$$

In the reference steady-state, the magnetic field $\mathbf{H}_0 = H_0\,\mathbf{l}_z$ and the magnetization $\mathbf{M}_0 = M_0\,\mathbf{l}_z$ are constant normal vectors with $M_0 = \chi\,H_0$ and $H_0 = H^{ext}/\left(1+\chi\right)$. We can thus define the potential $\phi = \nabla H$. Following a very common approach [6, 7, 14], we solve the magnetic field problem independently from the temperature field. Thus, for the non inductive case the Maxwell equations are reduced to the Laplace equation

$$\nabla^2\phi = 0. \tag{5}$$

In the non magnetic solid wall and in the gaseous phase outside the layer the potentials satisfy the same equation

$$\nabla^2\phi_S = 0 \quad \text{for} \quad z \le 0 \quad \text{and} \quad \nabla^2\phi_G = 0 \quad \text{for} \quad z \ge \zeta_\Sigma, \tag{6}$$

where $\zeta_\Sigma = d + \xi(x, y, t)$ is the local width of the layer defining the shape of the free surface.

The boundary conditions for the magnetic field at the wall ($z = 0$) and on the deformable surface ($z = \zeta_\Sigma$) are

$$\Delta\langle \mathbf{n} \times \mathbf{H} \rangle = \Delta\langle \mathbf{n} \cdot \mathbf{B} \rangle = 0, \tag{7}$$

where $\mathbf{n}$ is the unit vector directed from the liquid to the gaseous phase and $\Delta\langle f \rangle = f_G - f_L$ is a discontinuity jump of the function f at the interface.

Momentum and energy balance equations

For the non-inductive case, the motion of the magnetic liquid submitted to a magnetic field is described by the Navier-Stokes equations

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v}\right] = -\nabla p + \eta \nabla^2 \mathbf{v} + \rho \mathbf{g} \quad (\nabla \cdot \mathbf{v}) = 0, \tag{8}$$

where t is the time, $\mathbf{v}(u, v, w)$ is the velocity vector, ρ is the density, η is the viscosity, p is a generalized pressure obtained by adding to the usual hydrostatic pressure, the magnetostrictive pressure and the fluid-magnetic pressure [2, 6, 14]. Due to the adopted coordinate system, the gravity vector $\mathbf{g}$ is $(0,0,-g)$ when the layer rests on the rigid wall and $(0,0,g)$ for the layer hanging down from the ceiling.

The energy balance equation for a large number of ferrofluids can be written in the usual form

$$\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla)T = \kappa \nabla^2 T, \tag{9}$$

where κ is the thermal diffusivity. The boundary conditions are as follows. Considering the wall as a perfect heat conductor, no slip condition and constant temperature are assumed on it

$$\mathbf{v} = 0, \quad T = T_w = const \quad \text{at} \quad z = 0. \tag{10}$$

One has the following force balance at the free surface ($z = \zeta_\Sigma$) [2, 14]

$$\mathbf{n} \cdot \left[p + \frac{1}{2}\mu_0 (\mathbf{M} \cdot \mathbf{n})^2 - 2\mathrm{H}\sigma \right]\mathbf{I} = -\sigma \mathbf{n}(\nabla.\mathbf{n}) + [\nabla - \mathbf{n}(\mathbf{n} \cdot \nabla)](\nabla\sigma + \mathbf{T}^{\mathbf{m}} + \mathbf{T}^{\mathrm{v}}), \tag{11}$$

where $2\mathrm{H} = -\left(\frac{\partial^2 \zeta_\Sigma}{\partial x_1^2} + \frac{\partial^2 \zeta_\Sigma}{\partial x_2^2}\right)$ is the surface curvature, $\mathbf{T}^{\mathrm{v}}(\tau_{ik})$ is the Newtonian viscous tensor whose components are $\tau_{ik} = \eta\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right)$ and $\mathbf{T}^{\mathbf{m}}(T_{ik})$ is the magnetic stress tensor with components $T_{ik} = -\left\{p + \frac{1}{2}\mu_0 H^2\right\}\delta_{ik} + B_i H_k$, δ_{ik} is the Kronecker symbol and $(x_1, x_2, x_3) \equiv (x, y, z)$. The normal component of Eq. (11) is the balance between the total pressure, capillary pressure and magnetic normal traction while its tangential component expresses the balance between the thermocapillary, viscous and magnetic forces along the interface. Newton's law for heat transfer across the free surface is assumed:

$$-\lambda\left[\mathbf{n}\cdot\nabla T\right] = a\left(T - T_G\right), \tag{12}$$

where λ is the thermal conductivity of the liquid, a is the heat liquid-gas transfer coefficient and T_G is the gas temperature. The interface is expressed by the linearized kinematic equation

$$\frac{\partial \zeta_\Sigma}{\partial t} = w. \tag{13}$$

The reference steady-state

The reference steady-state corresponds to a motionless, heat conductive case where the temperature field is a linear function of the *z* coordinate:

$$T = T_w - \beta z, \tag{14}$$

with $\beta > 0$ for heating the layer from the wall side and $\beta < 0$ for heating it from the gaseous phase.

STATIONARY INSTABILITY OF FERROFLUID LAYER

The mechanical equilibrium of the ferrofluid layer can be destabilized due to two different mechanisms: interfacial magnetic instability and thermocapillary instability. Below the necessary conditions for the appearance of both instabilities will be found by the means of the stability theory.

The solution of the linearized system of equations and boundary conditions for the perturbations is searched in normal modes, representing the dependent variables in the form [15]

$$f = \delta f(z)\exp\left[\omega t + i\left(k_x x + k_y y\right)\right], \tag{15}$$

where $\delta f(z)$ is the perturbation amplitude, ω the growth constant and $k=\sqrt{k_x^2+k_y^2}$ the wavenumber. When $\omega = i\varpi$ is a pure imaginary number, the instability is oscillatory with a wave frequency $\varpi > 0$. For $\omega = 0$, the perturbations will not grow or decay and the instability is non-oscillatory or stationary. We consider underneath the response of our system to non oscillating linear disturbances.

Introducing d, d^2/κ, κ/d, βd, H_0 and $H_0 d$ to scale length, time, velocity, temperature, magnetic field intensity and magnetic potential, respectively, we study the equivalent dimensionless problem where we use the same notations for the dimensionless variables.

Magnetic field

Defining a differential operator $D = d/dz$, Eqs. (5) and (6) are reduced to the equations for the amplitudes of the potentials

$$\begin{aligned} &(D^2-k^2)\delta\phi = 0 \qquad \text{for} \qquad 0 \le z \le 1+\delta\xi, \\ &(D^2-k^2)\delta\phi_G = 0 \qquad \text{for} \qquad z \ge 1+\delta\xi, \\ &(D^2-k^2)\delta\phi_S = 0 \qquad \text{for} \qquad z \le 0. \end{aligned} \tag{16}$$

As the dimensionless amplitude $\delta\xi$ is small compared to one, all boundary conditions at the perturbed free surface are completed from the ones at $z=1$ by Taylor expansions in $\delta\xi$. Thus, the boundary conditions (7) are written as

$$\delta\xi = \frac{\delta\phi - \delta\phi_G}{1+\chi}, \qquad D\delta\phi = k\left(\delta\xi - \frac{\delta\phi}{1+\chi}\right) \tag{17}$$

at the free surface ($z=1$) and

$$\delta\phi = \delta\phi_S, \qquad D\delta\phi - \frac{k\,\delta\phi}{1+\chi} = 0 \tag{18}$$

at the wall ($z=0$). The magnetic potentials $\delta\phi_G$ and $\delta\phi_S$ should vanish far from the ferrofluid layer. The amplitude of the potential in the layer is given by [14]

$$\delta\phi = \delta\xi(1+\chi)\frac{(2+\chi)\exp kz + \chi\exp(-kz)}{(2+\chi)^2\exp k - \chi^2\exp(-k)}. \tag{19}$$

Normal velocity and temperature

The equations for the amplitudes of the normal velocity and temperature are given by

$$
\begin{aligned}
&(D^2-k^2)^2\delta W=0,\\
&(D^2-k^2)\delta T+\delta W=0.
\end{aligned} \tag{20}
$$

The boundary equations at the rigid wall are $\delta W = D\delta W = \delta T = 0$. Along the free surface, the normal and tangential stress balances (11) become

$$
\begin{aligned}
&Cr\,(D^2-3k^2)D\delta W - k^2\left(\pm Bo + k^2 - N_m\, k\Lambda(k)\right)\delta\xi = 0,\\
&(D^2+k^2)\delta W + Ma\,k^2\left(\delta T-\delta\xi\right)=0,
\end{aligned} \tag{21}
$$

where $\Lambda(k)=\dfrac{(2+\chi)\exp k-\chi\exp(-k)}{(2+\chi)^2\exp k-\chi^2\exp(-k)}$.

The dimensionless form of Eqs. (12) and (13) gives

$$
D\delta T + Bi\left(\delta T-\delta\xi\right)=0, \qquad \delta W = 0 \qquad \text{at} \quad z=1. \tag{22}
$$

The following dimensionless parameters are introduced: the Marangoni number $Ma=\gamma\beta d^2/\rho\eta\kappa$, the magnetization parameter $N_m=\mu_0(1+\chi)\,\chi^2 H_0^2 d/\sigma_0$, the Crispation number $Cr=\rho\eta\kappa/\sigma d$, the Bond number $Bo=\rho g d^2/\sigma_0$ and the Biot number $Bi=ad/\lambda$. The Marangoni number stands for the thermocapillary effect and the Bond number compares the displaced fluid weight $\rho g d$ to the capillary pressure σ_0/d. The positive or negative sign in the front of Bo corresponds to a layer resting on the wall or hanging from the ceiling, respectively. The parameter N_m, introduced in [14], is similar to the Bond number as it stands for the ratio of the magnetic force $\mu_0\,\mu M_0^2=\mu_0(1+\chi)\,\chi^2 H_0^2$ exerting on the interface, to the capillary pressure and therefore, it can be called *magnetic Bond number*. This number is always positive nevertheless the direction of the magnetic field, while the Marangoni number assumes positive or negative values for heating the layer from the wall side or through the free surface, respectively.

A non-trivial solution exists when the physical parameters involved satisfy the compatibility condition

$$
Ma=\frac{8k\left(\cosh k\sinh k-k\right)\left(k\cosh k+Bi\sinh k\right)}{\sinh^3 k-k^3\cosh k+\dfrac{8Crk^5\cosh k}{\pm Bo+k^2-N_m k\Lambda(k)}}. \tag{23}
$$

This equation coincides with the similar expression obtained in [10] if the magnetic Bond number N_m is replaced by the quantity $(1+\chi)Si\sqrt{Bo}$ where the parameter $Si=\mu_0\chi^2H_0^2/\sqrt{\rho g d}$ is defined by Bashtovoi *et al*. [6, 9, 10, 11]. The last parameter stands for the ratio of the surface magnetic force to the hydrostatic pressure and follows from Rosensweig and Cowley's condition (1) [1]. Note that in the absence of the magnetic field, Eq. (23) reduces to Takashima's equation for pure Marangoni instability in a thin liquid layer with a deformable free surface [16].

The compatibility condition (23) represents the Marangoni number as a function of the wavenumber at given values of the other parameters including N_m which reflects the variation of the magnetic field. As in experiments the magnetic field can be changed independently of the heat conditions, it is possible to keep the last ones constant and to vary the magnetic flied slowly. For such physical situation the presentation of N_m as a function of the other parameters is more convenient and Eq. (23) is transformed to

$$N_m = \frac{\pm Bo + k^2}{k\Lambda(k)} - \frac{8Ma\,Cr k^4 \cosh k}{\Lambda(k)\left[Ma\left(\sinh^3 k - k^3 \cosh k\right) - 8k\left(\cosh k \sinh k - k\right)\left(k\cosh k + Bi \sinh k\right)\right]}. \quad (24)$$

This formula is similar to the general expression given by Weilepp and Brandt [14] for the case of thermogravitational instability of the layer submitted to a magnetic field. If the ferrofluid is isothermal ($Ma = 0$), one obtains the condition for the Rosensweig instability of a finite ferrofluid layer sandwiched between semi-infinite magnetically inert phases:

$$N_w = \frac{\pm Bo + k^2}{k\Lambda(k)} \quad \text{or} \quad \pm Bo + k^2 - k\Lambda(k)N_m = 0. \quad (25)$$

This equation with positive sign in the front of the Bond number is discussed in [14].

RESULTS AND DISCUSSION

We study the neutral curves $Ma(k)$ and $N_m(k)$ using the physical properties of ferrofluid EMG 901 [14]. Let the ferrofluid layer resting on the solid wall, have a width equal to 1mm (which is less than the capillary length [14]), so that $Bo = 0.508$, $Cr = 10^{-5}$ and $Bi = 3\times10^{-4}$. Using these values, the critical Marangoni number in the pure *thermocapillary* case ($N_m = 0$) is $Ma_c^T = 79{,}6085$ at $k_c^T = 1.992$. The minimum of the neutral curve $N_m(k)$ for *surface magnetic static instability* in the isothermal case ($Ma = 0$) is $N_m^c = 4.981576$ at $k_c^m = 0.778$. The thermocapillary and the surface magnetic critical dimensionless wavelengths are very different and therefore, can be distinguished in experiments.

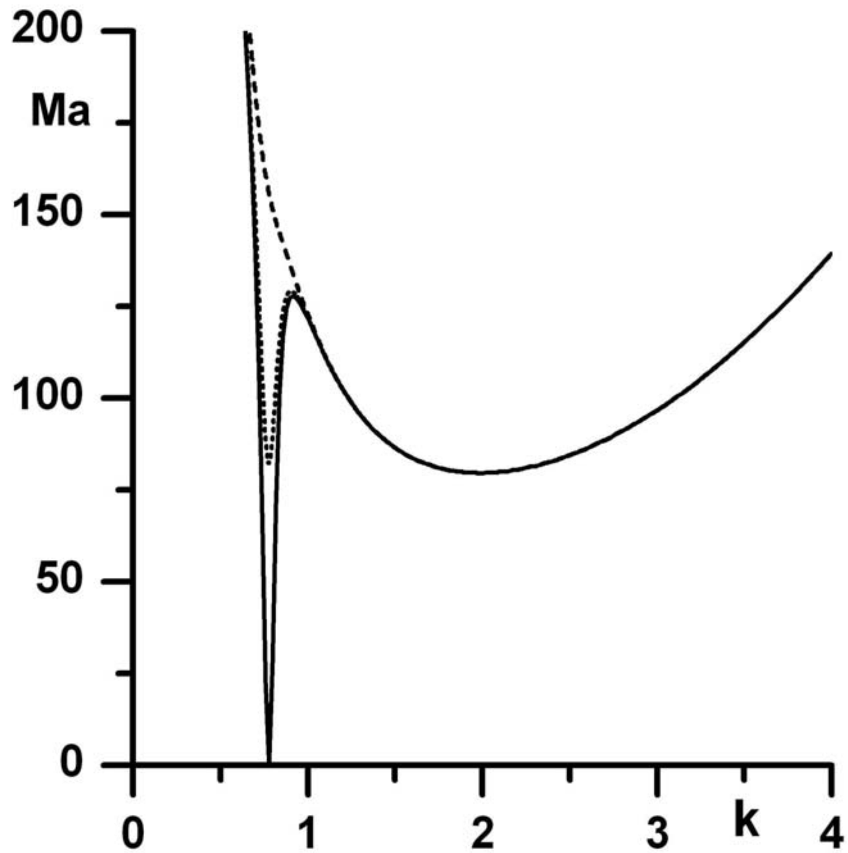

Fig. 1. Neutral curves $Ma(k)$ for values of $N_m = N_m^c$ (solid line), $N_m = N_m^{CD2}$ (dotted line) and $N_m = 4.9$ (dashed line)

Suppose that the magnetic field is constant (at given N_m) and the layer is heated gradually by increasing the temperature gradient. Neutral stability curves $Ma(k)$ for three values of N_m are plotted in Fig. 1. The curves for which $N_m \leq 4.9$ does not differ considerably from the one determined by Takashima's formula [16] (Eq. (23) for $N_m = 0$). Their shape is similar to the marginal curve for $N_m = 4.9$ (composed by the dashed line and the upper solid one). Those curves have one minimum at $k = k_c^T$ that decreases very slowly when increasing N_m up to $N_m = 4.9$. The magnetic field does not influence the thermocapillary instability of the layer.

However, for $N_m > 4.9$, a dip appears on the neutral curve at a wavenumber close now to k_c^m. For a relatively small increase of the magnetic Bond number, the dip deepens very quickly so that at $N_m^{CD2} = 4.97294 = 0.9983 N_m^c$ the system reaches a stationary co-dimension-2-point (CD2), i.e. the neutral curve presented by the dotted line has a minimum at $k = k_c^m$ equal to the other minimum $Ma_c^{CD2} = 79{,}5919 = 0.9998 Ma_c^T$ at $k = k_c^T$. Pavlinov [10] predicted the existence of such a point on the neutral curve $Ma(k)$. Further increase of N_m leads to decreasing the local minimum (at $k = k_c^m$) which becomes zero for $N_m = N_m^c$ (solid line). The thermocapillary instability has not the possibility to intervene since the static instability has taken over. Except for values of N_m and Ma very near to N_m^{CD2} and Ma_c^{CD2}, when there is a coupling between both instability mechanisms, for any other values of these parameters one of the mechanisms prevails.

For $N_m > N_m^c$, the local minimum at $k = k_c^m$ becomes even negative and decreases rapidly with even very small increase of the magnetic Bond number. For example, for $N_m = 4.99 = 1.0017 N_m^c$ the minimum is equal to -752.6. As for negative Marangoni numbers large enough in absolute value there is no solution of the problem, one may conclude that a quite strong heating from above could compete the action of the magnetic field (corresponding to a given value of the magnetic Bond number) and could keep the layer stable. But, the analysis shows that the layer may be destabilized by further increasing the magnetic field intensity. This result is in accordance with Schwab's experiments [12, 13] detecting instability for heating from both sides of the layer.

For N_m a little higher than a value of 4.99, the shape of the curve $Ma(k)$ differs drastically from the previous ones as it has two discontinuities (vertical asymptotic lines) which divide the curve into three branches. By increasing the wavenumber from zero, the first branch goes to minus infinity when k approaches the first asymptotic line at about $k_1 = 0.1$ (this point moves right when N_m increases). In the interval between this line and the next one at about $k_2 = 0.8$ the curve has a positive branch with one minimum. The second branch goes to infinity when the wavenumber k from (k_1, k_2) approaches the boundaries of this interval. For $k_2 < k$, the third branch increases from minus infinity to plus infinity. Practically, for any negative Marangoni number the solution of the stability problem exists. Therefore, in the case of heating from the gaseous phase, magnetic fields may destabilize the layer overwhelming the stabilizing thermocapillary effect.

For non-zero Crispation number, function (23) doesn't go to infinity when $k \to 0$ and has a finite value. For very small wavenumbers $(k << 1)$, the Marangoni number (23) is expressed by the following formula obtained as Maclaurin expansion,

$$\lim_{k\to 0} Ma = \frac{2(1+Bi)}{3\,Cr}\left\{ \begin{array}{l} \pm Bo - \dfrac{k\,N_W}{2\,(1+\chi)} \\ + k^2\left[1 \mp \dfrac{3\,Bo}{10} - N_W\,\dfrac{\chi(2+\chi)}{4\,(1+\chi)^2} - \dfrac{Bo^2}{120\,Cr} \pm \dfrac{Bo(3+Bi)}{6(1+Bi)}\right] \end{array} \right\}. \tag{26}$$

This function is a parabola which, due to the coefficient multiplying k^2, is convex from below for relatively small Crispation numbers and concave for larger values of Cr. The numerical analysis shows that for the abovementioned values of the parameters, Eq. (26) is a good approximation of Eq. (23) for $k < 0.015$ only.

Under some assumptions, Pavlinov [10] (see also [6]) derived a simplified asymptotic formula of (23) written in our notations as

$$\lim_{k \to 0} Ma = \frac{2(1+Bi)}{3\,Cr}\left[Bo - \frac{k\,N_W}{2\,(1+\chi)} + k^2 \right]. \qquad (27)$$

At $k_* = N_m / 4(1+\chi)$, this parabola has a minimum which becomes zero for $N_m^* = 4(1+\chi)\sqrt{Bo}$ ($Si^* = 4$). In the case considered above ($Bo = 0.508$) the curve touches the abscissa at $k_* = \sqrt{Bo} = 0.713$ for $N_m^* = 6.557$. Although this wavenumber is, by occasion, close to the threshold wavenumber $k_c^m = 0.778$ for the Rosensweig instability, the use of (27) for predicting the surface magnetic instability is absolutely wrong because the asymptotic formula is valid for very small wavenumbers ($k < 0.005$). As was already shown, the neutral curve (23) has a zero minimum for $N_m^c = 4.981576$ corresponding to the value $Si = 3.039$ which is different from $Si^* = 4$ as Pavlinov found. One may conclude that the simplified formula for $Ma(k)$ predicts wrong physical results as it is used incorrectly for much larger values of k.

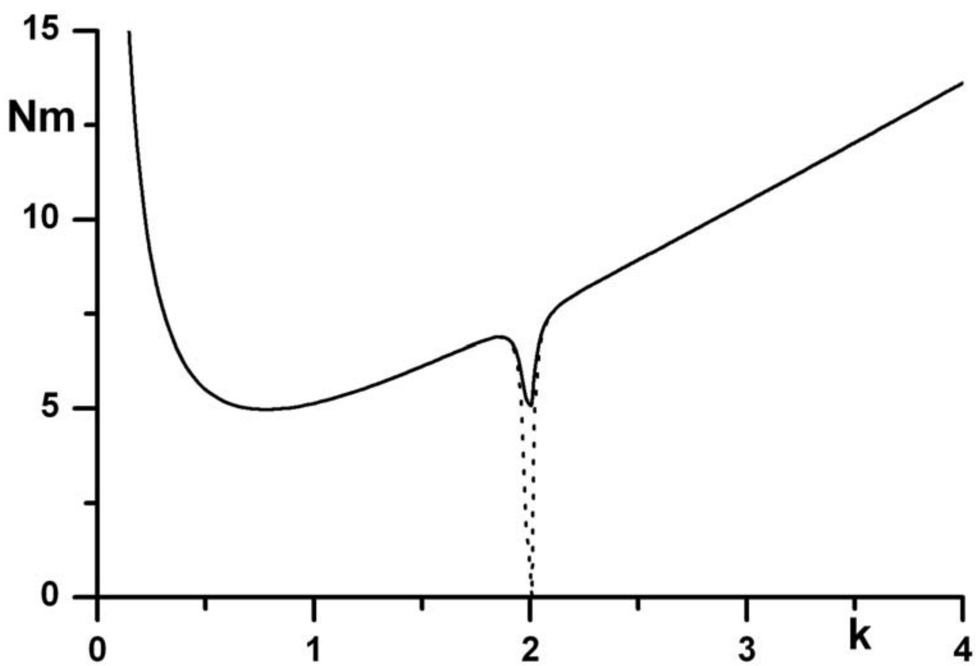

Fig. 2. Neutral curves $N_m(k)$ for values of $Ma = Ma_c^{CD2}$ (solid line) and $Ma = Ma_c^T$ (dotted line)

This conclusion about the incorrect result of Pavlinov [10] in predicting the interfacial magnetic instability threshold is important not only for the thermocapillary case considered here. Unfortunately, his idea to use a simplified asymptotic form of the compatibility condition for much larger wavenumbers than those for which it is valid, was also applied in other cases of thermocapillary [11] and thermogravitational [9, 6] instability where incorrect results for the stability criteria were also obtained.

Consider now the case when the magnetic field intensity is fixed and the temperature gradient is increased slowly by augmentation of the temperature of the solid plane. Taking into account thermocapillary and gravitational effects, Weilepp and Brandt [14] studied curve $N_m(k)$ for different Marangoni and Rayleigh numbers. Their results for small Rayleigh numbers do not differ from ones for pure thermocapillary case. Here, we will show results for zero Rayleigh number.

Marginal curves $N_m(k)$ from (24) are displayed in Fig. 2 for two representative values of the Marangoni number. For $Ma = Ma_c^{CD2}$, the solid curve has again two equal minima at $k_c^m = 0.778$ and $k_c^T = 1.992$ which are equal to N_m^{CD2}. For $Ma = Ma_c^T$, the neutral curve presented by the dashed line touches the abscissa at $k = k_c^T$ (pure thermocapillary case). It is seen again that the coupling between the thermocapillary and magnetic mechanisms is possible for a pair of values of the Marangoni number and the magnetic Bond number for which waves of two different lengths are expected to occur simultaneously. For $Ma > Ma_c^{CD2}$ the neutral curve $N_m(k)$ crosses the abscissa. As the negative values of N_m have no physical meaning, the values of k for which $N_m < 0$ do not belong to the set of possible unstable wavenumbers.

CONCLUSIONS

We study the coupling of the Marangoni and the Rosensweig instabilities in a thin horizontal ferrofluid layer bounded by a solid wall plane and open to a gaseous phase from the other side. The layer is submitted to a vertical weak magnetic field and can be heated from each side. As the critical wavelengths of both instabilities are quite different, the appearance of each of them can be easily observed in experiments. We restrict ourselves to the non-inductive case to obtain the necessary conditions for the appearance of both instabilities separately and their coupling.

The linear stability analysis is developed for the stationary case giving an analytical solution of the problem. The compatibility condition is derived in two equivalent forms representing either the Marangoni number or the magnetic Bond number as a function of the wavenumber. Neutral curves of both types are plotted. It is shown that a coupling between both instabilities exists in a very restricted zone of the parameter space. For other values, the coupling between the instabilities disappears completely and one of them is only physically relevant. We derive an asymptotic formula for the compatibility condition for very small wavenumbers and show that the simplified form of this formula, used in the literature, leads to incorrect physical results.

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